November 1, 1971

FRA-TM-22

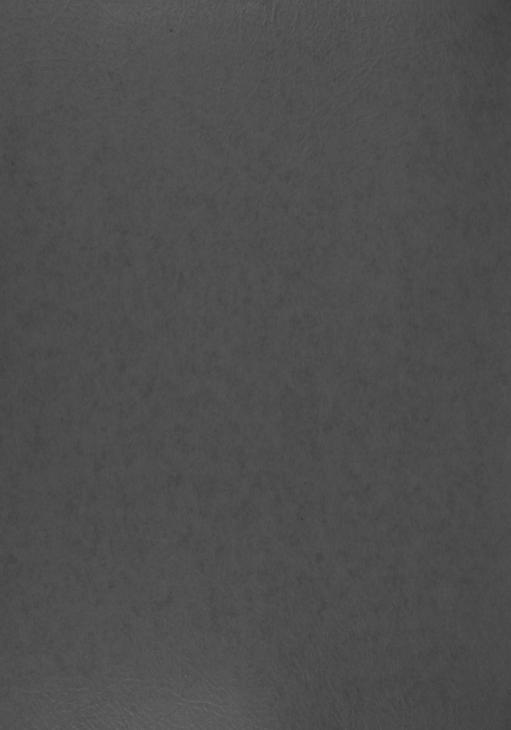
BUCKLING IN FAST REACTORS. I.

B. I. Spinrad

Argonne National Laboratory Applied Physics Division 9700 South Cass Avenue Argonne, Illinois 60439

FRA TECHNICAL MEMORANDUM NO. 22

Results reported in the FRA-TM series of memoranda frequently are preliminary and subject to revision. Consequently they should not be quoted or referenced without the author's permission.



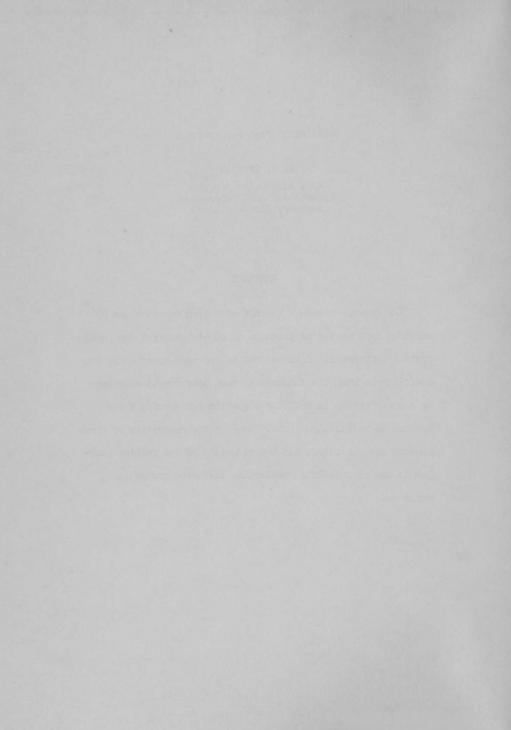
Buckling in Fast Reactors. I.

B. I. SPINRAD

Applied Physics Division Argonne National Laboratory Argonne, Illinois 60439

ABSTRACT

The energy-dependent reactor diffusion equation can be expanded in a series of products of purely spectral and purely spatial components. A convenient set of base spectra are the coefficients when the fundamental mode spectrum is expressed as a power series in B^2 . The algorithm for forming these spectra is relatively simple. Some of the properties of these spectra are exhibited, and the reduction of the reactor problem to one of definable overlapping diffusion groups is explored.



INTRODUCTION

The concept of buckling was introduced in the first explorations of reactors for several reasons:

- (1) The reactors were large and almost all of their volume was describable by the fundamental spatial and spectral mode.
- (2) Reflector savings were relatively small, and the buckling provided an accurate estimate of critical dimensions.
- (3) With infinite multiplication only slightly greater than unity, measurement of buckling, combined with estimation of the form of P(B) from experiment or theory permitted more accurate determination of k_{∞} than any other technique.

The first reactors were large, natural-uranium-fueled ones, to which these comments are very pertinent. However, the early fast reactors were small, highly reflected, and with k_{∞} in the core around 2. To such reactors, the concept of buckling was not useful, and the main theoretical approaches emphasize transport and multigroup calculations, with the fundamental mode only one of many important spatial modes in the reactor.

However, the relative simplicity of buckling measurements prompted experimental groups, first at Los Alamos and then at Argonne¹ to attempt to determine buckling and bring its measurements into consonance with the theoretical models available. A particularly large effort was expended in determining the (negative) buckling of an assembly of natural uranium.²

As the design of fast reactors for central-station power generation evolved, it became apparent that these reactors were reverting toward systems for which the buckling could again be a useful parameter. The

PETRODOCTICS

The concept of bushling was immuniced in the rions applications of

son lerrors to the special property and the special state of the special state of the special special

toolstend to the client elevistic to arealize areas as believed to arealize areas as believed to areas and areas are also areas areas are also are also areas are also are also are also areas are also areas are also are also areas are also a

measurement of buckling, comined with estimation of the four of P/1) —
from experiment on theory — permitted more accurate directal principal of the theory other techniques.

The first resoture were large, addingnouncement thereof dress to reduce these comments are very pairtheunt. Hanever, the durp test reaching were small, identity reflected, and with by in, the one around 2. To such transfer tone, the concept of building was not module, and the main despreaded.

Hosever, the relative simplicity of bus ing measurement a supplied of the Alares and then at Argone to attempt to determine budding and being its nessenteeds into commonwealth the forestrical models available. A curticularly large effort was equalisated in determining ine (negative) budding of an escably of natural available.

evolved, if brains apparent that these resulters here evanting reserved evolved for which the buddling could again to a matul parameter. The

incorporation of large amounts of ^{238}U into the core to facilitate internal breeding brought values of k_{∞} into the range 1.0-1.5. Even at the high power densities of breeders, the size of reactor cores to supply heat for 1000 MW of electrical generation becomes large enough so that we expect major portions of the core to be only mildly influenced by reflector transients. With understanding of the behavior of reflector savings, the estimation or measurement of buckling then becomes a useful predictor of criticality in fast reactors. Thus, Hummel was able, through the ELMOE code, 3 to use changes in buckling of reactors to separate reactivity effects which are intrinsic in the core from those which are reflectorinduced. In France, there have been a number of experiments on buckling, both to determine core critical properties and to use buckling as a measure of reactivity difference in substitution experiments. 4

Up to now, the interpretation of buckling experiments has been primarily a matter of comparing measured values with those deduced from such basic calculations as ELMOE. Where discrepancies appear, the cross-section set itself must bear the burden. This is entirely proper in the ultimate; however, the discovery of adjustments to nuclear data is much eased if a simpler interpretation of buckling can lead to another, easily characterized integral parameter. In thermal reactors, as had been mentioned, this interpretation is provided by the relation

$$k_{\underline{M}}P(B) = 1, \tag{1}$$

where P(B) is the nonleakage probability associated with buckling B^2 . Equation (1) holds for critical systems, which is a strong experimental condition. In thermal reactors, P(B) is at least partly measurable, since it is related to migration during slowing down, which is measurable

incorporation of these secures of ""direct we care to technical infermal inequiar brought values of a into the cases large sample. Incorporate high power densiries of biradomy, the cites of rescrice cares common to be incorporated for the direct sample of the direct major for the cases of the cases of

Up to now, the interpretation of budding experiments has been mixmurity a number of comparing averaged values with those deduced from such
basic calculations as ELYES. More discuspaniles appear? The expensional features the budget, This is entirely proper in the dislinator,
however, the discovery of caltermonia to number that is such easer if a
stagion interpretation of budding can less to enother, pastly charactended integral parameter. In themsel associate, as lest been mentioned,
this integral parameter. In themsel associate, as lest been mentioned,

2 2 (0)7, 3

where P(B) is the nonlegacy projection accordated with thickling B.
Equation (I) holds for critical systems, which is a strong conscioustal
condition. In themselves resource, P(B) is no least purely measurable,
since it is related to nignation during storing day, initial in westernia

and only slightly perturbed by absorption, and to thermal neutron migration which is either accurately described by diffusion theory in the global sense, or is small. The standard interpretation of P(B) is as the Fourier transform of the spatial distribution of flux leading to absorption, from a point source of fission neutrons in an infinite medium whose slowing-down and absorption characteristics are identical with those of the reactor, but in which fission does not occur (Weinberg⁵).

This (correct) interpretation has discouraged attempts to estimate P(B) for fast reactors. It is well-nigh impossible to mock up a fast reactor core for migration measurements, and simultaneously to suppress fission. Thus, estimation of P(B) is limited to the examination of the variation of B^2 with, for example, ν , in ELMOE-type codes. This procedure is again correct; however, it suffers from the fact that inference of P(B) for negative or complex B^2 , which is often desirable to estimate reflector-induced transients, is a very imprecise extrapolation.

The first portion of this paper is the presentation of a method for deriving P(B) as a power series in B^2 , using purely spectral calculations. The significance of this technique lies not only in its ability to explore P(B) for negative or complex B^2 , but also in the fact that the spectra associated with powers of B^2 have in themselves interesting properties which suggest the utility of overlapping multigroup formalisms and Lie series for calculating global properties of fast reactors.

A POWER SERIES FOR P(B)

Assuming diffusion theory, the space-energy equations for any reactor may be written as:

and only alightly perturbed by discribed and to release in the last to related and the state of the leading to describe the state of the sound of the state of the leading to describe the state of the

FOR for fact reaction. It is well-righ impossible to more up a feat.

Western care for adgration resourcements, and simultaneously, to adopted

thesian. Thus, estimation of F(S) is limited to the scannain of the

western of S: with, for manuals, v. in district the society. This proofdure is again correct; consven, it suffers from the fact that interests
of F(S) for mention or couples S: which is cites desirable to entirest

The limit post of this paper is the presentation of a method for deriving P(B) as a power parties in B?, using parely appoints calculations. The significance of this recinique him also in the fact that the appoint the recipies or conflict B?, but also in the fact that the appoint as sentimed with parents of B? have in themselves interesting provintions which suggest the whility of overlapping and lignoup femalisms and like sentime for calculating plots is properties of fast coactors.

A POMER SERVING FOR P(E)

Andredes different topory, the speck-among equations for moy really top many be weitten out:

$$-D(E)\nabla^{2}\phi(\overset{\rightarrow}{\mathbf{r}},E) + \left[\Sigma_{\mathbf{a}}(E) + \Sigma_{\mathbf{s}}(E)\right]\phi(\overset{\rightarrow}{\mathbf{r}},E) - \int_{E}^{\infty}\Sigma_{\mathbf{s}}(E^{\prime})\phi(\overset{\rightarrow}{\mathbf{r}},E^{\prime})P_{\mathbf{s}}(E^{\prime} \to E) \ dE^{\prime}$$

$$= \chi(E) \int_0^\infty v \Sigma_f(E^*) \phi(r,E^*) dE^*. \qquad (2)$$

The symbols in Eq. (2) have their canonical meanings. The only unusual ones are $P_s(E' \to E)$, the probability that a scattering event for a neutron of energy E' will lead to its emergence at energy E, and $\chi(E)$, the fission spectrum. If there are several fissionable isotopes, $\nu \Sigma_f$ is interpreted as Sum $\nu_i \Sigma_{fi}$ for the "i" isotopes. In addition to the assumptions of diffusion theory, Eq. (2) assumes that scattering is isotropic, which is virtually within diffusion theory, and that the fission spectrum is invariant to fissioning species. This latter assumption is almost correct if there is a dominant fissionable material, and is easily set right by defining a $\bar{\chi}$; it is made only to simplify notation.

Suppose there exists a fundamental mode and a critical reactor. Then the fundamental mode may be written

$$\phi_0(\vec{r}, E) = f_0(\vec{r}, B_0) S_0(E) ,$$
 (3)

where

$$\nabla^2 f_0(\vec{r}, B_0) = -B_0^2 f_0(\vec{r}, B_0) . \tag{4}$$

Substitution of Eqs. (3) and (4) into Eq. (2) permits factorization and division of f_0 , and leads to

$$\begin{bmatrix} B_0^2 D(E) + \Sigma_a(E) + \Sigma_s(E) \end{bmatrix} S_0(E) - \int_E^{\infty} \Sigma_s(E') S_0(E') P_s(E' \to E) dE'$$

$$= \chi(E) \int_0^{\infty} v \Sigma_f(E') S_0(E') dE'. \qquad (5)$$

-31 (d - -31 9(-3,5) (-32) - (3,5) (3) 3 * (3) 3 * (3) 3 * (3) 3 * (3) 3 * (3) 4 * (3) 5 * (3) 4 * (3) 5 * (3) 6 * (3

"35 ("3,510(13), 20 (3));

The symbols in Eq. (2) have their canceled meanings. The only through ones are F (E - E), the probability that a scattering sweet for a search of every E will lead to its emergence at energy E, and *(E). The Fischer of energy E, and *(E). The Fischer of energy E, and *(E). The first one as we were a first instinct on the auturation to the upper that scattering is instruction to the care. Fq. (2) assumed that scattering is instruction to the instruction theory, Eq. (2) assumed that scattering is instruction to fingioning apacies. This latter assumption is above to invariant to fingioning apacies. This latter assumption is above correct if there is a during this isomable material, and is easily set

Suppose there exists a fundamental mode and a triffical reactor.

(3),2(,0,0),1 = (2,7),0

wastk.

C. 8, 57, 1381 + C. 8, 57, 159

Substitution of Eqs. (3) and (4) into Eq. (2) permits factorisation and division of for and leads to

m (a - , m acm acm 2] - (m s (m) 1 + (m) 2 + (m) 2

If we integrate Eq. (5) over all E, remembering that $\chi(E)$ is normalized to unity and that the integral of P_S over emergent energies is also unity, we get:

$$\int_{0}^{\infty} \left[B_{0}^{2} D(E) + \Sigma_{a}(E) \right] S_{0}(E) dE = \int_{0}^{\infty} v \Sigma_{f}(E') S_{0}(E') dE'.$$
 (6)

Useful relations may also be derived for an idealized infinite-medium experiment. If an infinite medium has a unit fission-spectrum source per unit volume, then the spectrum of neutrons would be given by S_{∞} , where S_{∞} is the spectrum which solves:

$$\left[\Sigma_{a}(E) + \Sigma_{s}(E)\right] S_{\infty}(E) - \int_{E}^{\infty} \Sigma_{s}(E') S_{0}(E') P_{s}(E' + E) dE' = \chi(E) . \tag{7}$$

Integration over all energies reveals that since the neutron production rate, f χ dE, is unity, so is the absorption rate, f $\Sigma_a S_\infty$ dE. If we now ask what the production rate would be if the spectrum were permitted to cause fission, it would be f $\nu \Sigma_f S_\infty$ dE. Thus,

$$k_{\infty} = \int_{0}^{\infty} v \Sigma_{f} S_{\infty} dE .$$
 (8)

The fact that systems with small bucklings have spectra approaching those of infinite media is well known. It suggests that we look for solutions of S_0 in the form of

$$S_0 = \sum_{n=0}^{\infty} (-B_0^2)^n G_n$$
 (9)

with

$$G_0 = S_{\infty} . ag{10}$$

If we integrate Eq. (5) over all E, recomberlos that x(2) is none.

Lizzed to unity and that the integral of E, over correct courses is also unity, we get:

Useful relations may also be derived for an idealised infinites, medium experiment. If an infinite medium has a unit the aparence would be given by source for unit volume, then the apartnum of neutrons would be given by 8, where 2 is the spectrum which colves:

Integration over all enoughes reveals that sinds the number production rate, I 2,5 of. If ... we have see what the production rate would be if the spectrum were peralread to cause fination, it would be I vz.2 of. Thur.

The fact time systems with small bucklings have spectra approximations of infinite main in well known. It suggests that we look for solutions of S. in the form of

(0.0)

Substitution of Eq. (9) into Eq. (5) yields:

$$\sum_{n=0}^{\infty} (-B_0^2)^n \left\{ \left[\Sigma_{\mathbf{a}}(E) + \Sigma_{\mathbf{s}}(E) \right] G_{\mathbf{n}}(E) - \int_{E}^{\infty} \Sigma_{\mathbf{s}}(E') G_{\mathbf{n}}(E') P (E' \to E) dE \right. \\ \left. - \chi(E) \int_{0}^{\infty} v \Sigma_{\mathbf{f}}(E') G_{\mathbf{n}}(E') dE \right\} = \sum_{n=1}^{\infty} (-B_0^2) D(E) G_{\mathbf{n}-1}(E) . \tag{11}$$

We now define G by the relation

$$\left[\Sigma_{\mathbf{a}}(\mathbf{E}) + \Sigma_{\mathbf{a}}(\mathbf{E})\right] \mathbf{G}_{\mathbf{n+1}}(\mathbf{E}) - \int_{\mathbf{E}}^{\infty} \Sigma_{\mathbf{S}}(\mathbf{E}') \mathbf{G}_{\mathbf{n+1}}(\mathbf{E}') \mathbf{P}_{\mathbf{S}}(\mathbf{E}' + \mathbf{E}) d\mathbf{E}' = \mathbf{D}(\mathbf{E}) \mathbf{G}_{\mathbf{n}}(\mathbf{E}) . \tag{12}$$

Substituting Eq. (12) into Eq. (11) gives:

$$\begin{bmatrix} \Sigma_{\mathbf{a}}(\mathbf{E}) + \Sigma_{\mathbf{s}}(\mathbf{E}) \end{bmatrix} G_{0}(\mathbf{E}) - \int_{\mathbf{E}}^{\infty} \Sigma_{\mathbf{s}}(\mathbf{E}') G_{0}(\mathbf{E}') P_{\mathbf{s}}(\mathbf{E}' \to \mathbf{E}) d\mathbf{E}'$$

$$= \chi(\mathbf{E}) \int_{0}^{\infty} v \Sigma_{\mathbf{f}}(\mathbf{E}') \sum_{\mathbf{n}=0}^{\infty} (-B_{0}^{2}) G_{\mathbf{n}}(\mathbf{E}') d\mathbf{E}'. \tag{13}$$

Since $G_0 = S_{\infty}$, we may substitute Eq. (7) to get

$$1 = \sum_{n=0}^{\infty} (-B_0^2) \int_0^{\infty} v \Sigma_f(E') G_n(E') dE'.$$
 (14)

From Eq. (8), we next get

$$k_{\infty} \sum_{n=0}^{\infty} (-B_{0}^{2})^{n} \left[\int_{0}^{\infty} v \Sigma_{\mathbf{f}}(E^{*}) G_{n}(E^{*}) dE^{*} \right] / \left[\int_{0}^{\infty} v \Sigma_{\mathbf{f}}(E^{*}) G_{0}(E^{*}) dE^{*} \right] = 1 . \quad (15)$$

All that remains is to define

Substitution of Eq. (9) into Eq. (5) yields:

We now define G by the relation

Substituting Eq. (12) into Eq. (11) gives:

Since G = S, we may existinte Eq. (7) to get

From Eq. (3), we next get

All that remains is to defin

$$g_{n} = \int_{0}^{\infty} v \Sigma_{f}(E^{*}) G_{n}(E^{*}) dE^{*} / \int_{0}^{\infty} v \Sigma_{f}(E^{*}) G_{0}(E^{*}) dE^{*}, \qquad (16)$$

and to identify Eq. (15) with Eq. (1). This yields

$$P(B) = \sum_{n=0}^{\infty} g_n (-B^2)^n . (17)$$

(See footnote a.)

The above proof suggests an algorithm for computing G_n and g_n . We take as given a program which will solve for the infinite medium spectrum with a fission spectrum source. That is, we assume that we have means of solving Eq. (7); a subroutine of ELMOE will do as an example. Then, the G_n may be computed by successive substitution of D G_{n-1} for χ , and the g_n by direct quadrature. The result is a purely spectral calculation of P(B).

EXAMPLE: AN AGING SYSTEM

We may illustrate the process analytically by taking as an illustration a system with the following properties:

- (a) monoenergetic fission source at energy E0, lethargy 0;
- (b) slowing-down in a continuous mode without absorption to lethargy U; and
- (c) absorption and fission without further migration on reaching lethargy U.

These assumptions mean that Eq. (5) may be written as

$$B_0^2 D S_0 + \frac{\partial}{\partial u} \left[\xi \Sigma_s S_0 \right] = k_{\infty} \delta(u) \left[\xi \Sigma_s S_0 \right]_{II}, \qquad (18)$$

and to identify Eq. (15) with Eq. (1). This visite

(See footpote a.)

The above proof suggests an algorithm for computing G_{ij} and g_{ij} be take as given a program which will solve for the individual matter spectrum source. That is, we assume that we have means of solving G_{ij} (7); a subroutine of EIDS will do so an example. Then, the Solving G_{ij} may be computed by successive substitution of D G_{ij} for χ , and the g_{ij} by direct quadrature. The result is a purely spectral calculation of P(8)

EXAMPLE: AN ACTING SYSTEM

We may illustrate the process analytically by taking as on illustration a system with the following properties:

- (a) monomergetic fission source at every Es. lathersy is
- of mattyreds most be about accommon a oil most-gaineds (d)
- (c) absorption and fission without further migration on moudhing to lather U.

There assumptions were that Eq. (5) may be swiften as

$$\mathbf{B}_{0}^{2}\mathbf{D}.\mathbf{S}_{0}+\frac{2}{8u}\left[\mathbf{E}\mathbf{E}_{0}\mathbf{S}_{0}\right]:=\mathbf{k}_{\infty}\delta\left(\mathbf{u}\right)\left[\mathbf{E}\mathbf{E}_{0}\mathbf{S}_{0}\right]_{U}$$

(81)

where we have converted to lethargy space, and need only consider u in (0,U). Equation (7) becomes

$$\frac{\partial}{\partial u} \left[\xi \Sigma S_{\infty} \right] = \delta(u) . \tag{19}$$

Then,

$$G_{0} = \frac{1}{\xi \Sigma_{S}},$$

$$G_{1} = \frac{1}{\xi \Sigma} \int_{0}^{u} \frac{D \, du'}{\xi \Sigma_{S}(u')},$$

$$G_{2} = \frac{1}{\xi \Sigma_{S}} \int_{0}^{u} \frac{D}{\xi \Sigma_{S}} (u') \, dU' \int_{0}^{u'} \frac{D}{\xi \Sigma_{S}} (u'') \, du'',$$
(20)

Etc.

Neutron age is given by

$$d\tau = \frac{D \ du}{\xi \Sigma_{S}}, \qquad \tau(0) = 0, \qquad (21)$$

so that Eqs. (20) become:

$$G_0 = \frac{1}{\xi \Sigma_s} \Big|_{\tau}, \qquad G_1 = \frac{1}{\xi \Sigma_s} \Big|_{\tau}, \qquad G_2 = \frac{1}{\xi \Sigma_s} \Big|_{\tau} \frac{\tau^2}{2}.$$
 (22)

In general,

$$G_{n} = \frac{1}{\xi \Sigma_{c}} \bigg|_{\tau} \frac{\tau^{n}}{n!} . \tag{23}$$

Further,

where we have converted to letherny space, and need only consider u in

mort

$$\frac{1}{23} = \frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right) = \frac{1}{2} =$$

Stee.

fertiron age is given by

$$\frac{D \cdot dt}{d\tau} = \frac{D \cdot dt}{\tau} = \tau 0$$

so that Eqs. (20) become:

$$\frac{2y}{60} = \frac{1}{62a} + \frac{1}{2} = \frac{1}{62a} + \frac{1}{2} = \frac{1}{60} = \frac{1}{60}$$

In general,

Telleria.

$$\int_{0}^{\infty} v \Sigma_{f} G_{n} du = k_{\infty} \left(\xi \Sigma_{S} G_{n} \right)_{\tau(U)} . \tag{24}$$

Therefore,

$$g_{n} = \frac{\tau(U)^{n}}{n!}$$
 (25)

and

$$P(B) = \sum_{0}^{\infty} (-B^{2})^{n} \frac{\tau(U)^{n}}{n!} = \exp \left\{-B^{2}\tau(U)\right\} . \tag{26}$$

This is, of course, the classical value.

A COMMENT ON THE G-SPECTRA OF FAST REACTORS

Typical fast reactors have essentially three spectral regions:

- (1) At high energies, the spectrum is dominated by the interplay between the fission-spectrum source and inelastic scattering events involving large energy losses. Absorption is adequately represented by a smooth curve, both because of the extreme validity of the narrow resonance approximation and the smearing action of Doppler broadening on resonances.
- (2) At intermediate energies, roughly 5-100 keV, the spectrum can be calculated quite well as a superposition of a gross spectrum, generated from continuous slowing-down theory with effective smeared absorption, and detailed resonance calculation.
- (3) At lower energies, the continuous slowing-down model remains useful, but one must treat some resonances (such as those in sodium) in detail.

Most of the need for representation of the spectrum in many groups arises from the high-energy groups. In principle, continuous slowing-down should be adequate to describe energy transfer at the lower energies.

Describera

5 cm

$$\{\alpha_{1}, \alpha_{2}, \alpha_{3}, \alpha_{4}, \alpha_{5}, \alpha_{5},$$

Into is, of course, the classical value.

A COMMENT ON THE C-SPECTER OF PAST REACTORS

Typical fast reactors have essentially those speciful regions:

- (1) At high energies, the spectrum is deminated to the interplay hetered the fission expectrum source and inelastic mathematics events involving large energy larges energy large. Absorption is absquately represented by a smooth curve, both because of the extrang validity of the narray recontroc approximation and the spectrum settion of Doppler broadshing on resolution.
- (2) At intermediate energies, roughly 5-100 keV, the spectrum was be calculated quite well as a superposition of a gross spectrum, generated from continuous slowing-down theory with effective mestry shortfies, and defailed resonance calculation.
 - (3) At lower onergies, the continuous slowing-down model results useful, but one must treat some reconsences (such as those in socium) in detail.

Nort of the need for representation of the spectrum in way grants arises from the high-energy groups. In principle, continuous elasing-

Now, if we note the trend of the G-spectra for the aging system, we observe that each ${\sf G}_{n+1}$ is <u>softer</u> than the previous ${\sf G}_n$. We expect this to hold true for fast reactors as well. In essence, then, we may expect that, after the first few ${\sf G}_n$, we can switch to a much simpler mathematical format to derive higher ${\sf G}_j$; solution of the continuous slowing-down equations involves simple quadrature in u. Ultimately, the ${\sf G}_n$ should approach highly peaked functions which may be well approximated analytically.

THE ADJOINT FUNCTION

The same procedure as recommended for the flux may be used to determine an adjoint expansion.

The adjoint equation analogous to Eq. (5) is

$$\begin{bmatrix} B_0^2 D(E) + \Sigma_a(E) + \Sigma_s(E) \end{bmatrix} S_0^{\dagger}(E) - \Sigma_s(E) \int_0^E S_0^{\dagger}(E') P(E \to E') dE'$$

$$= \nu \Sigma_f(E) \int_0^\infty \chi(E') S_0^{\dagger}(E') dE'. \qquad (27)$$

For the infinite system, the adjoint spectrum is the solution of

$$\left[\Sigma_{a}(E) + \Sigma_{s}(E)\right] S_{\infty}^{T}(E) - \Sigma_{s}(E) \int_{0}^{E} S_{0}^{+}(E') P_{s}(E \rightarrow E') dE' = \nu \Sigma_{f}(E) . \tag{28}$$

We may write a system of equations such that:

$$G_0^{\dagger} = S_{\infty}^{\dagger}$$

$$\left(\Sigma_a + \Sigma_s\right)G_{n+1}^{\dagger} - \Sigma_s \int_0^E G_{n+1}(E^{\prime})P_s(E + E^{\prime}) dE^{\prime} = D G_n$$
(29)

Then we may write

Now, if we note the trend of the G-spectra for the aging system of cheerys that each C_{0.1} is soiter than the previous G₀. We expect this to hold true for fast reactors as well. In excess, then, we may expect that, after the first few G₀, we can exited to a much simpler mathematical format to derive lighter C₁: solution of the continuous slowing-days special format to derive lighter C₁: solution of the continuous slowing-days special through the first few days are the specially the G₁ should approach the first few days and the special functions which may be well approximated shally intelly

THE ADJOING PUNCTION

The same procedure as recommended for the flux may be used to datermine an adjoint expansion.

The adjoint equation analogous to Eq. (5) is

For the infinite system, the adjoint spectrum is the solution of

(32)
$$= \frac{1}{2} \left[\frac{1}{2} \left(\frac{1}{2} \right)^2 \left($$

We may write a system of equations such that:

Then we may write

$$S_0^+ = \sum_{n=0}^{\infty} (-B_0^2)^n G_n^+, \qquad (30)$$

provided that

$$\int_{0}^{\infty} \chi(E') \sum_{0}^{\infty} (-B_{0}^{2})^{n} G^{+}(E') dE' = 1$$
 (31)

is satisfied.

Let us multiply Eq. (28) by $G_0(E)$ — or $S_\infty(E)$ — and integrate over energy. This leads to the following equation after reversal of integration order and replacement of S_∞^+ by G_0^+ :

$$\int_0^\infty G_0^+(E) dE \left(\Sigma_a + \Sigma_s\right) G_0 - \int_E^\infty \Sigma_s G_0(E') P_s(E' \to E) dE' = \int_0^\infty v \Sigma_f(E) G_0(E) dE.$$
(32)

Substituting Eq. (7) for the term in braces in Eq. (32),

$$\int_0^\infty G_0^+ \chi \ dE = \int v \Sigma_f^- G_0 \ dE = k_\infty \ . \tag{33}$$

We may thus define a critical condition

$$k_{\infty}P^{+}(B^{2}) = 1$$
, (34)

where

$$P^{+}(B_{0}) = \int_{0}^{\infty} (-B_{0}^{2})^{n} g_{n}^{+}$$

$$g_{n}^{+} = \int_{0}^{\infty} \chi(E) G_{n}^{+} dE \int_{0}^{\infty} \chi(E) G_{0}^{+} dE$$
(35)

It is well known that the buckling spectrum of flux and adjoint are identical. From this, it follows that

, pr(8a-> } - 3

serir fishlyong

r = rab (m'o'(h-) (cmx)

de seriefied.

Let us multiply Eq. (28) by G_g(E) — or S_g(E) — and imagrams owns werey. This leads to the following equation situs reversal of integration order and replacement of S² by C²;

Scharicuring Sq. (7) for the term in braces in Eq. (32).

to may thus define a critical condition

mesulti.

jattan j = tanta

It is well town that the buckling spectrum of filts and aljudel according

(00)

The adjoint function has the property of orthogonality to the real flux spectrum in the sense that:

If B_n^2 are the several values of B^2 which satisfy the critical equation [(17) or (34)]; and if

 B_n^2 are all different.

Then

$$\int_{0}^{\infty} S_{n}(E)S_{m}^{+}(E)D(E) dE = 0 , \quad \text{for } m \neq n ,$$
 (37)

APPLICATION OF THE G-SPECTRA

The properties of the G spectra which can be applied stem from their (assumed) completeness and from the form of expansion of the fundamental mode. If the G_n are a complete set (which, except for pathological cases, they are), then they may perhaps be of use in defining alternate multigroup systems to the usual step-function-in-energy system. In particular, such a system might have the first group as fundamental mode, with other group spectra orthogonal to it. Then it would be possible to deal with transients, as introduced by reflectors and boundaries, more directly.

MULTIGROUP FORMULATIONS

In a multigroup model of standard type, $\phi(\vec{r},E)$ is replaced by an n-vector, $\vec{\phi}(\vec{r})$, where the "n" components of ϕ represent the flux in

(2) 2 - (2) 2

(41)

The edjoint firstlen has the property of orthogonality to the rest flux spectr win the same than

If By are the marked values of B enich satisfy the critical squared then [(10)] or (10)]; and if

describite like one to

nshir

APPLICATION OF THE G-SPECTIV

The properties of the C spectra which can be applied atom that teaming the features) completes and from the form of expansion of the Condensated mode. If the C are a complete set (which, except for rational place team, they are), then they may perhaps be of use, in defining alternate multiproup systems to the usual oten-tunotion-in-energy eyetem. In perturbat, such a system might have the first group as fundamental mode, with other group as fundamental mode, with other group as fundamental mode, with other group as fundamental mode, with transferred spectrum of the first transferred as introduced by reflectors and boundaries, more directly.

MILTE CROUP FORMULATIONS

In a militaroup model of standard type, e(r.E) is replaced by an newector, I(r), where the "a" components of a represent the Elem in

specified E-regions, D and Σ_a are diagonal matrices, integration over $\nu\Sigma_f$ is replaced by premultiplication by an n-adjoint-vector, $\chi(E)$ becomes a vector X, and the scattering law is replaced by a transfer matrix.

The fundamental mode has a spectrum described by an eigenvector, and these are n-l other eigenvectors which represent the higher-mode spectra. B_n^2 is often complex for n>0, and the higher modes have complex spectra; however, n-l linearly independent real spectra, all of which are orthogonal to the fundamental mode, can be synthesized from them.

The critical equation in an n-group system is always expressible in the form

$$k_{\infty} = \sum_{k=0}^{n} a_k B^{2k} . \tag{38}$$

This means that the criterion for good representation of any system by a multigroup scheme might by that 1/P(B) should be a series which terminates after a certain number of powers of B^2 .

The most direct method of determining this is by formal inversion of P(B). We may write

$$1/P(B) = \sum_{k=0}^{\infty} a_k B^{2k}$$
, (39)

and derive the a iteratively from

$$a_{0} = 1
a_{k} = \sum_{\ell=1}^{k} (-)^{\ell+1} a_{k-\ell} g_{\ell}$$
(40)

Because a_k and g_ℓ are dimensional, and because of cumulation of errors, it is recommended that convergence be examined by inspecting $a_{n+1}a_{n-1}/a_n^2$; if this number suddenly becomes small, an n-group approxi-

mation is justified. If, after many terms have been examined, there seems to be an approach to a constant ratio, an age-like solution can be suspected, and examination of the convergence of a_k to $\alpha T^k/k!$, where α and T are constants, is recommended.

If we have an N group system, G_N and higher will be linear combinations of G_0 , . . G_{N-1} . Thus, all necessary relations can be obtained by computing 2N functions — N fluxes and adjoints, or 2N fluxes, which is algorithmically simpler.

Note that for a true N-group system, all a_n for n > N must vanish. Thus, one should examine $a_{n+2}a_{n-2}/a_n^2 {\rm etc.}$, for a few terms before concluding that an N-group approximation is valid.

A mathematically more satisfactory way of determining the validity of an N-group approach is to determine the degree to which χ/D may be represented by a linear combination of $G_0 \cdot \cdot \cdot G_{N-1}$. If this is the case, G_{N-1} is a linear combination of χ/D and g's of lower order, whence G_N et seq. also are (from the recursion relations for G).

The equivalent statement is that G_N would likewise be a linear combination of $G_0 \dots G_{N-1}$. A necessary condition for this to be the case is the vanishing of the determinant of g's to order 2N:

$$\begin{bmatrix} g_0g_1 & \cdot & \cdot & g_N \\ \cdot & \cdot & \cdot \\ g_N & \cdot & \cdot & g_{2N} \end{bmatrix} = 0 .$$

This effectively states that there is a linear combination, P, of ${\rm G_0}$. . . ${\rm G_{N-1}}$ such that

mation is justified. If, after many name have been commissed, there per to be an appearance of the convergence of a to of Ad. where a said T are constraints, is recommended.

Note that for a true M-group system, all a, for a all must verish.

Thus, one should examine a_{nvelop-2}/a₀sto., for a few terms before conclusting that an M-group supprecimentum is valid.

A communically more satisfactory way of determining the validity of an M-group approach is to determine the depole to which x/D may be represented by a linear combination of G₀ . . . G_{0,1} . If this is the case, see a linear combination of x/D and g's of lower order, whence C₀ et seq. also are from the recursion relations for G).

9 - 1208 0 = 1208

This effectively mates that there is a linear combination, P. of

$$\int_0^\infty G_m^\dagger D \Big(G_N - P \Big) dE = 0 \qquad m = 0, \dots N-1$$
and
$$\int_0^\infty v \Sigma_f \Big(G_N - P \Big) dE = 0.$$
(42)

It also means that $a_{n+1} \cdot \cdot \cdot a_{2N}$ all vanish. In other words, if the critical equation of form (39) has any terms of order greater than B^{2N} , they start at B^{4N+2} . While not mathematically impossible, such cases must be classified as pathological.

For comparison, the N-th order determinant for an age theory system would be of the order of:

$$\left(g_{1} \right)^{N^{2}+N} \left(\prod_{0}^{N} n! \right) / \left(\prod_{N}^{2N} n! \right) .$$
 (43)

The determinant in Eq. (41) would have to be small compared to expression (43) in order to justify an N-group theory.

Once an N group theory has been established, we have some liberty in defining the groups which we shall use. All of them are, of course, linear combinations of the ${\sf G}_n$.

The obvious choice is to define our groups orthogonal to each other, and to take as our first group the fundamental mode. To express it in terms of G_0 . . . $\mathsf{G}_{\mathsf{N-1}}$ (remembering that G_{N} et seq. are themselves linear combinations of them), we start with the relation

$$g_{N+k} = -\sum_{\ell=1}^{N} (-)^{\ell} g_{N+k-\ell} a_{\ell}$$
, (44)

(1-11 0 = m. a. = ma (1 - p) d a a

bas

0 = 30 (9 - ,0) 30

It also means that a_{nel} . . . a_{pp} all vanish. In other words, if the ordinosit equation of form (49) has any terms of order present than a set of the order words. While not mathematically impossible, such cases must be classified as paradogical.

for emperison, the k-th enter determinant for an age theory switch would be of the order of:

[10]] \ [10]] min[13]

(64)

The determinant in Eq. (41) would have to be enail compared to expression (43) in order to justify an N-proup theory.

once on a group theory has been established, we have some liberty to defining the groups which we shall use. All of them are, of course.

c 152-4+112³(-) 12 - c 3+113

(44)

which follows from (40) and the vanishing of a_{ℓ} for $\ell > N$. It may now be shown that this corresponds to a similar expression for the G's:

$$\sum_{n=0}^{N} (-)^{n} a_{n+k-n} = 0 , \qquad (k > 0) .$$
 (45)

We now take Eq. (9), multiplying S_0 by k_∞ on the left-hand side and the infinite sum in Eq. (9) by the series expression for k_∞ in terms of B_0^2 [Eq. (38) with n = N). The use of Eq. (45) then leads to:

$$S_{0} = \frac{1}{k_{\infty}} \sum_{n=0}^{N-1} (-)^{n} G_{n} \sum_{\ell=n}^{N-1} a_{\ell-n} B_{0}^{2\ell}$$

$$= \frac{1}{k_{\infty}} \sum_{\ell=0}^{N-1} B_{0}^{2\ell} \sum_{n=0}^{\ell} (-)^{n} a_{\ell-n} G_{n} .$$
(46)

Other groups may be defined to illustrate the transient spatial behavior of specific reactions. We may choose a group for one reaction which is orthogonal to the fundamental; a group for another which is orthogonal to the first two; and so on until we have N-2 reactions illustrated. Then the first reaction has a spatial dependence derived from the fundamental mode and the first of these synthetic groups; the spatial dependence of the second reaction depends on these two, plus the next synthetic group; and so on. (With the fundamental mode and N-2 such transients, we have a "ghost" group left over; at least one such "ghost" is needed in order to preserve group orthogonality.)

THE FISSION TRANSIENT

We illustrate the formation of groups by looking at a particularly straightforward one to synthesize. This is that transient which union fortices from (80) and the vanishing of a, for 5 × 8. It may him he show that that the conversation for the 9's:

$$(0 + 0) = 0 + 0 = 0$$

We now take Eq. (9), multiplying S, by k, on the lair-band side and the initialize aum in Eq. (9) by the series expression for k, in terms of S (Eq. (53) with n = 10). The use of Eq. (45) that leads to:

Other groups may be defined to illustrate the transient opailed behavior of specific respicance. We say crosse a group for one readshow which
is crumiqued to the fundamental; a group for another which is arbitrated,
to the first precision has a special departure derived from the fundamental
transfer precision has a special departure derived from the fundamental
mode and the filter of these synthetic groups; the special departures of
the second reaction departure on these two, plus the next synthetic group;
and so on. (With the fundamental tests and N-2 such transients, we have a
send so on. (With the fundamental tests and N-2 such transients, we have a
send so on. (With any fundamental tests and N-2 such transients, we have a
send so on those one of the such "givest" is needed in order to

THE PESTON TRANSITATI

distributions as a property of groups by looking at a particularity of the translation that is that translation on one property than it.

incorporates all the fission rate which is not part of the fundamental mode. We look for a spectrum of the form $\alpha S_0 - \beta \chi/D = S_f$ which is orthogonal to S_0 . Then, all other spectra which are linearly independent of S_0 and S_f will be orthogonal to them [in the f A[†](E)D(E)B(E) dE sense]. It follows that only S_0 and S_f will have nonvanishing integrals over $\nu \Sigma_f$.

If we now adopt the notation:

$$\langle AB \rangle \equiv \int_0^\infty A^{\dagger}(E)D(E)B(E) dE$$
, (47)

the desired function is

$$\left\langle S_0 \left| \frac{\chi}{D} \right\rangle S_0 - \left\langle S_0 S_0 \right\rangle \frac{\chi}{D} = S_f.$$
 (48)

The representation of χ/D as a function of the \boldsymbol{G}_n is now necessary. Formally, we may write

$$\frac{X}{D} = \sum_{0}^{\infty} X_{n}G_{n} \tag{49}$$

$$\gamma \equiv \left\langle \frac{\nu \Sigma_{f}}{D} \frac{\chi}{D} \right\rangle , \qquad (50)$$

and obtain the x_n by solution of:

Equation (51) is operationally difficult to solve, but conceptually

incorporates all the firston rate which is not part of the fundamental mode in the dependence of the forms S_0 . Then, all other spectra which are linearly independent of S_0 and S_0 . Then, all other spectra which are linearly independent of S_0 and S_0 will be undopped to them (in the J A (E)D(E)B(E) dE sensel. It follows that only S_0 and S_1 will have nonvanishing integrals over v_{L_1} .

II we now adopt the notations

at notions benjest sit

The representation of χ/D as a function of the G_n is now necessary outside, we say write

and obtain the x by solution of

Equation (51) is operationally difficult to solve, but concept and the most solve and the the the determinant of the matrix in Eq. (51) is, for a Chife

approximation, the same one as is used to test the validity of a multigroup approximation of finite order.

THE MULTIGROUP EQUATIONS

We finish this first part of the specification of the method by seeing how we construct multigroup equations. Let us assume that we have completed the construction of S_0 , S_f . . ., etc., in such a fashion that each one is orthogonal to the others. Let us further assume that we have, by hook or crook, arranged for the multigroup representation to be finite (this is a convenience rather than a necessity). Finally, let us label $S_0 \equiv Q_1$, $S_f = Q_2$. . . up to Q_N . Thus, I have a set of

$$Q_{i} = \sum_{j=1}^{N} C_{ij} G_{j-1}$$
, (52)

where all the C; have been defined. Similarly,

$$Q_{i}^{T} = \sum_{j=1}^{N} C_{ij} G_{j-1}^{+}$$
 (53)

and

$$\langle Q_i Q_j \rangle = 0$$
, $i \neq j$ (54)

We now write

$$\phi(\vec{r}, E) = \sum_{k=1}^{N} Q_{k}(E) f_{k}(\vec{r}) , \qquad (55)$$

upproximation, the same one as is used to test the validity of a salitgroup approximation of fights order.

PROTTAGE STORETCHE

We finish this first part of the specification of the method by seeing has we construct multigroup equations. Let us assume that we have consisted the construction of S_p. S_p. T. T. etc., in sort a fastion that each one is orthogonal to the others. Let us further assume that we have, by book or crowlence for the multigroup representation to be finite (this in a convenience rether than a morestity). These let us label Sp = Q_p.

where all the C. have been defined. Similarly,

. 神色素一种

t x 2 (0,0)

We now twite

(2),2(3),0 1 = (3,5).

(22)

and substitute into Eq. (2),

$$-\sum_{k=1}^{N} Q_{k}(E)D(E)\nabla^{2}f_{h}(\vec{r}) + \sum_{k=1}^{N} f_{k}(\vec{r}) \left\{ \left[\Sigma_{a}(E) + \Sigma_{s}(E) \right] Q_{k}(E) \right.$$

$$- \int_{E}^{\infty} \Sigma_{s}(E^{\prime})P_{s}(E^{\prime} + E)Q_{k}(E^{\prime}) dE^{\prime} \right\}$$

$$= \chi(E) \sum_{k=1}^{N} f_{k}(\vec{r}) \int_{0}^{\infty} \nu \Sigma_{f}(E^{\prime})Q_{k}(E^{\prime}) dE^{\prime}. \tag{56}$$

Multiplying by Q_1^+ and integrating (remember, $Q_1 = S_0$) gives

$$-\left\langle Q_{1}Q_{1}\right\rangle \nabla^{2}f_{1}+\sum_{k=1}^{N}f_{k}(\vec{r})\left[\left\langle \frac{v\Sigma_{f}}{D}Q_{k}\right\rangle -DB_{0}^{2}\left\langle Q_{1}Q_{1}\right\rangle \right]=\sum_{k=1}^{N}f_{k}\left\langle \frac{v\Sigma_{f}}{D}Q_{k}\right\rangle ,\tag{57}$$

which reduces to

$$(\nabla^2 + B_0^2)f_1 = 0$$
,

as it should.

Multiplication by Q2 and integrating gives:

$$-\left\langle Q_{2}Q_{2}\right\rangle \nabla^{2}f_{2} + \sum_{k} f_{k}^{\lambda_{2k}} = -\frac{\left\langle Q_{2}Q_{2}\right\rangle}{\left\langle Q_{1}Q_{1}\right\rangle} \left[f_{1} - \frac{\left\langle Q_{2}Q_{2}\right\rangle}{\left\langle Q_{1}Q_{1}\right\rangle} f_{2}\right]. \tag{58}$$

The λ_k are obtained by substituting the expansions for Q_n [into Eq. (52)] into Eq. (57), using the recursion relations for G_k , and substituting back the resulting expressions from G into Q.

Similar expressions are found by integrating over Q_n^+ for n > 2. The result is a set of N equations, of which the first, as fundamental mode,

negatiful as set of W equations, of which the first, as fundamental mode,

is independent of the rest. The other N-1 equations represent a full matrix of equations with sources to the set coming from the fundamental mode. The Laplacian term is purely diagonal. The set is therefore determinate.

Each spectrum defines a \mathbf{D}_k — an effective diffusion coefficient — such that $\mathbf{D}_k \nabla \mathbf{f}_k$ is continuous across normal boundaries. Also, each \mathbf{f}_k is continuous (these properties derive from the detailed spectral continuities of the original equation). In general, it will prove impossible to satisfy all boundary conditions unless the reactor is critical.

REFLECTORS

Reflectors are, in general, nonmultiplying; therefore the χ terms will be small or vanish. Reflectors also have a different scattering maxtrix from cores, and different absorption.

The equations for the reflector must be obtained as in the core.

It is assumed that the same set of orthogonal spectra describe the entire problem. Then the reflector equations become:

$$-\left\langle Q_{\mathbf{k}}Q_{\mathbf{k}}\right\rangle \nabla^{2}\mathbf{f}_{\mathbf{r}\mathbf{k}}+\sum_{k=1}^{N}\mathbf{f}_{\mathbf{r}\mathbf{k}}\int_{0}^{\infty}\frac{\mathbf{D}_{\mathbf{c}}(\mathbf{E})Q_{\mathbf{k}}}{\mathbf{D}_{\mathbf{r}}(\mathbf{E})}+d\mathbf{E}\left[\boldsymbol{\Sigma}_{\mathbf{a}_{\mathbf{r}}}+\boldsymbol{\Sigma}_{\mathbf{s}_{\mathbf{r}}}-\int_{\mathbf{E}}^{\infty}\boldsymbol{\Sigma}_{\mathbf{s}}Q_{\mathbf{k}}P_{\mathbf{s}}(\mathbf{E}^{*}\rightarrow\mathbf{E})\right]d\mathbf{E}^{*}$$

$$= \int_{0}^{\infty} \chi \frac{D_{c}(E)}{D_{c}(E)} Q_{k} + (E) \sum_{\ell=1}^{N} f_{\mathbf{r}k} \int_{0}^{\infty} \nu \Sigma_{fr} Q_{\ell}(E') dE'.$$
 (59)

The appropriate integrals must be evaluated to find a set of group equations for the $\mathbf{f}_{\mathbf{rk}}$.

is independent of the rest. The other N-1 equations represent a full metrix of equations with sources to the ret coming from the fundamental mode. The Laplacian term is purely diagonal. The set is therefore determinate.

Lach spectrum delines a D, —, an effective diffusion observation. — such that D, W, is continuous across normal boundaries. Also, each t, is continuous (these properties darive from the detailed spectral one tinuities of the original equation). In general, it will prove impossible to satisfy all boundary conditions unless the reactor is critical.

KEFLECTORS

Reflectors are, in general, normaltiplying; therefore the x terms will be small or vanish. Reflectors also have a different scattering maximum from cores, and different absorption.

The equiptions for the reflector must be obtained as up the core.

It is essumed that the same set of orthogonal spectra deduction the entire problem. Then the reflector equations become:

(ea)
$$\frac{1}{2} \frac{1}{2} \frac{1}{2}$$

The appropriate integrals must be evaluated to find a set of group equations for the fac-

FOOTNOTE

^aA mathematically rigorous proof would require the demonstration that the G_n defined by Eqs. (10) and (12) form a complete set of functions in the space defined by solutions of Eq. (5). Our proof is equivalent to replacing the coefficient of χ in Eq. (5) by an arbitrary multiplier, assuming convergence of g_n , and identifying the multiplier as unity for a critical system. We think that it illustrates the physics better. Note also that we could have gotten to Eq. (4) directly by substituting Eqs. (9), (10), and (12) into Eq. (6).

The Og defined by Eqs. (10) and (12) form a complete set of Euchicos in the Og defined by Eqs. (10) and (12) form a complete set of Euchicos in the Space defined by solutions of Eq. (5). Our proof is equivalent to replacing the continuous of a in Eq. (5) by an architecture multipliers as make nor a continuous of E , and identifying the multiplier as under nor a continuous of E , and identifying the multiplier as under nor a continuous continuous for (1) directly by substituting .

REFERENCES

- BEYER, F. C., et al., "The Fast Exponential Experiment," Proc.
 Intern. Conf. Peaceful Uses of Atomic Energy, Geneva, 1958, Vol. 5
 (United Nations, New York), pp. 342-346.
- CHEZEM, C. G., "A Uranium-Metal Exponential Experiment," Nucl. Sci. Eng., 8, 652 (1960). This definitive paper cites most of the significant earlier work as well.
- 3. RAGO, A. L., and HUMMEL, H. H., "ELMOE: An IBM-704 Program Treating Elastic Scattering Resonances in Fast Reactors," Argonne National Laboratory Report ANL-6805 (1964). Needless to say the original ELMOE program has been translated to other machines and is now mostly available as a piece of other, larger programs.
- 4. BARBERGER, H., et al., "Analysis of Experiments Performed in MASURCA," in Proc. Symp. on the Physics of Fast Reactor Operation and Design, British Nuclear Energy Society (1969), pp. 25-39.

REPRESENTA

- 1. BEFER, E. C., et al., "The Pest Excondutial Experiment," Epoca Intern. South Peaceful Uses of Atomic Surryy, Sansur, 1988, vol. 6 Chifred Marians, New York), pp. 347-346.
- C. CHIMEN, C. C., "A Unsainte-Matel Exponential Experiment," Nucl. out. Awar, 8, 662 (1860). This definitive paper cires most of the significant earlier work as well.
- 2. PAGO, A. L., and HAMEL, H. H., "ILMOE: An IRM-TON Program Treating
 Elastic Scattering Resonances in fast Reactors," Argome Maricoss

 Laboratory Report ANL-6865 (1964). Needless to say the original

 ELMOE program has been translated to other machines and is now

 mostly available as a piece of other, larger programs.
- 4. BARLEWER, H., et al., "Analysis of Experiments Performed in MASIRCA,"
 in Proc. Emp. on the Physics of Fast Resover Operation and Design,
 British Mudlean Energy Society (1968), pp. 25-29.

